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Solutions to Written Exam at the Department of Economics summer 2019

Economics of Exchange Rates

May 28, 2019

Number of questions: This exam consists of 2 questions.

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1. **Exchange rate determination**

Consider the Mundell-Fleming model comprised of the following equations

$$\dot{s} = i - i^* \tag{1}$$

$$m = \sigma s + \kappa y - \theta i \tag{2}$$

$$\dot{y} = \chi \left(\alpha + \mu s - y \right) \tag{3}$$

where notation is standard.

(a) Explain the economic rationale behind these equations.

(b) Derive the two differential equations (the LM-curve and the IS-curve) and illustrate the model in the y-s plane.

(c) Show the effects of expansionary monetary policy on the exchange rate and output. Explain carefully!

(d) Show the effects of expansionary fiscal policy on the nominal exchange rate and output. Compare with the effects from monetary policy. Is fiscal policy effective? If so explain why.

(e) Consider then the model where we replace equation 3 with

$$\dot{p} = \gamma \left(\alpha + \mu \left(s - p \right) - \bar{y} \right) \tag{4}$$

such that the model the three equations (1), (2) and (16) comprise the model. What are the main differences between these two models? **Hint:** Replace s with p in equation (2).

(f) Derive the two differential equations (the goods market and the money market equilibrium curves) and illustrate the model in the s-p plane.

(g) What are the effects of expansionary monetary policy on the nominal exchange rate and the price level in this model? Compare and contrast these effects to the effects of monetary policy in the Mundell-Fleming model above.

2. **Micro-based macro model**

   (a) Explain the underlying assumptions of Evans micro-based macro model.

   (b) Demand for euros in week t by agent $n \in [0,1]$ is given by

$$\alpha_t^n = \alpha_s(\mathbb{E}_t^n \Delta s_{t+1} + \hat{r}_t - r_t) + h_t^n \tag{5}$$

   hedging demand is

$$h_t^n = \alpha_z z_t^n \tag{6}$$

   and the link between the microeconomic environment of agent n and the state of the economy is given by

$$z_t^n = z_t + v_t^n \quad \text{where} \int_0^1 v_t^n dn = 0 \tag{7}$$

   Show that the aggregate demand for euros can be written as

$$\alpha_t = \alpha_s(\overline{\mathbb{E}}_t^n s_{t+1} - s_t + \hat{r}_t - r_t) + h_t$$

   where h_t is the aggregate hedging demand and $\overline{\mathbb{E}}_t^n s_{t+1} = \int_0^1 \mathbb{E}_t^n s_{t+1}\, dn$.

   (c) Assume that the spot exchange rate quoted by dealers is given by

$$s_t = \mathbb{E}_t^D s_{t+1} + \hat{r}_t - r_t - \delta_t. \tag{8}$$

   Show that the risk premium can be written as

$$\delta_t = \mathbb{E}_t^D \left[s_{t+1}^e - \tfrac{1}{\alpha_s} h_t \right] \tag{9}$$

   where $s_{t+1}^e = s_{t+1} - \overline{\mathbb{E}}_t^n s_{t+1}$ under efficient risk sharing $\mathbb{E}_t^D \alpha_t = 0$. Provide an interpretation!

   (d) The spot exchange rate in this model is given by

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} + \tfrac{1}{\alpha_s} \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i h_{t+i} - \tfrac{1}{\rho} \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i s_{t+i}^e. \tag{10}$$

   Discuss the implications of this expression and how it relates to standard monetary models of the exchange rate.

   (e) Summarize the empirical evidence on the relationship between order flows, news announcements and exchange rates.

Solutions

1. This question relates to the following learning objectives. Skills: Describe the main models of exchange rate determination (the Monetary approach to the exchange rate, Dornbusch overshooting model, the portfolio balance model and Lucas asset pricing model) and apply these models to analyze the effects of monetary and fiscal policy on the exchange rate, and summarize the empirical evidence on these models. Describe and apply Mundell-Fleming models to analyze the effects of economic policy under both flexible and fixed exchange rates. Competences: Carry out economic analysis related to exchange rate determination, forecasting and international financial management.

   (a) We assume a small open economy, output is demand determined and prices are fixed. Equation (1) is the UIP relation. We assume that UIP holds and we assume rational expectations such that the change of the exchange rate $\dot{s}$ is equal to the interest rate differential between the two countries $i - i^*$.

   Equation (2) is the money market equilibrium condition where m is money supply (which is equal to money demand), y is output. The parameter κ is the output elasticity of money demand, θ is the interest elasticity of money demand and σ is the exchange rate elasticity of money demand. Note that this expression differs from the standard money demand function since we include s instead of p on the RHS. The reason for this is that we have an underlying assumption in the M-F model that prices are fixed (and we then normalize such that the log of the price levels in the domestic and in the foreign countries are both equal to zero). Then, the overall price level (comprised of the price on domestic goods and of imported goods measured in home currency) is a function of the exchange rate. The parameter σ measures the importance of foreign prices in the domestic consumer price index.

   Equation (3) is the aggregate demand for goods. The first component on the RHS of this equation (α) is the autonomous component of aggregate demand, the next component (μs) represents foreign demand for domestic goods, and the last component is output such that the change in aggregate demand also depends on output in the previous period.

   (b) To solve for the LM–curve: Assume that the monetary authority can set the money supply (it is given exogenously), then solving for i in equation (2) and insert this solution into (1) yields

   $$\dot{s} = \frac{\sigma}{\theta}s + \frac{\kappa}{\theta}y - \frac{1}{\theta}m - i^*. \qquad (11)$$

   The IS–curve: Use equation (2) to solve for i and insert into (3) such that

   $$\dot{y} = \chi\left(\alpha + \mu s - y\right). \qquad (12)$$

   The system can now be rewritten in matrix form

   $$\begin{bmatrix} \dot{s} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\sigma}{\theta} & \frac{\kappa}{\theta} \\ \chi\mu & -\chi \end{bmatrix} \begin{bmatrix} s \\ y \end{bmatrix} + \begin{bmatrix} -\frac{1}{\theta}m - i^* \\ \chi\alpha \end{bmatrix} \qquad (13)$$

where the determinant of the coefficient matrix is

$$-\chi\frac{\sigma}{\theta} - \frac{\chi\mu\kappa}{\theta} < 0$$

such that the system is saddlepath stable.

We can now obtain the LM curve (in the s and y plane) where the exchange rate market is in equilibrium ($\dot{s} = 0$) and the IS curve where the goods market is in equilibrium ($\dot{y} = 0$). The slope of the LM curve is negative and the slope of the IS curve is positive. To show this, note that the LM curve (first row of (13) and let $\dot{s} = 0$) is given by

$$s = -\frac{\kappa}{\sigma}y + \frac{\theta}{\sigma}\left(\frac{1}{\theta}m - i^*\right)$$

such that

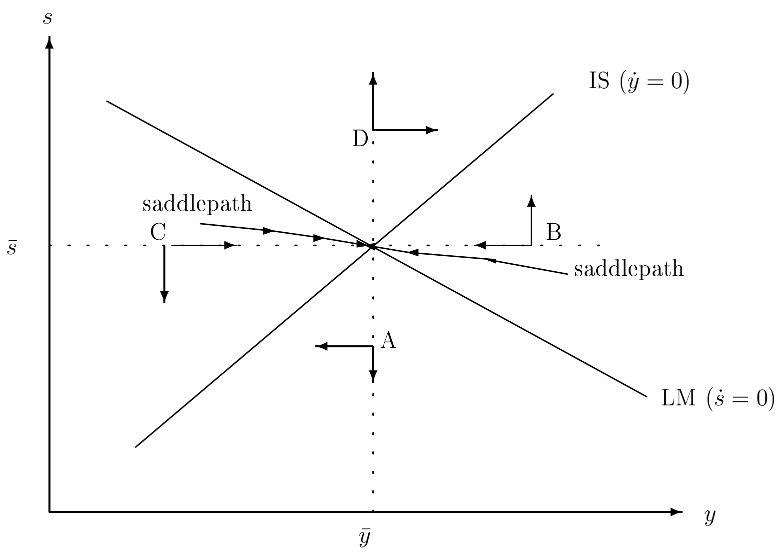
$$\frac{\partial s}{\partial y} = -\frac{\kappa}{\sigma}$$

and the IS curve (second row of (13) and let $\dot{y} = 0$) is given by

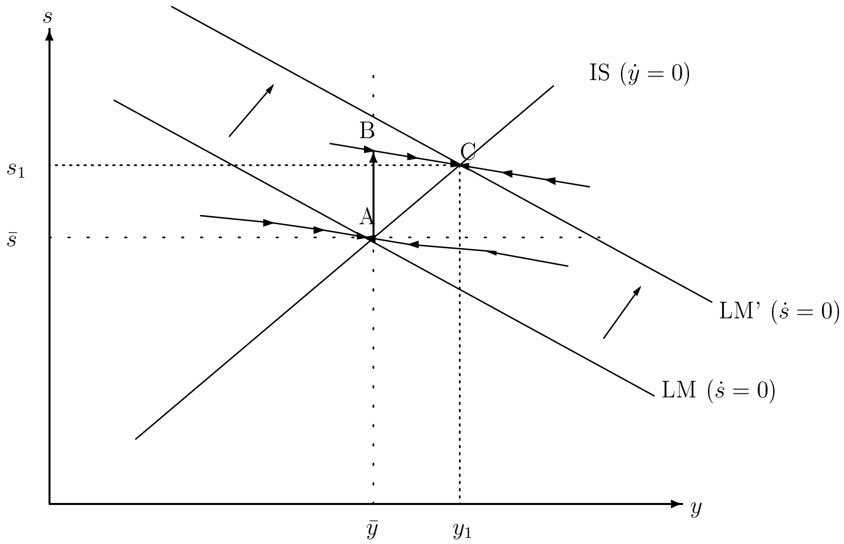
$$s = \frac{1}{\mu}y - \frac{\alpha}{\mu}$$

such that

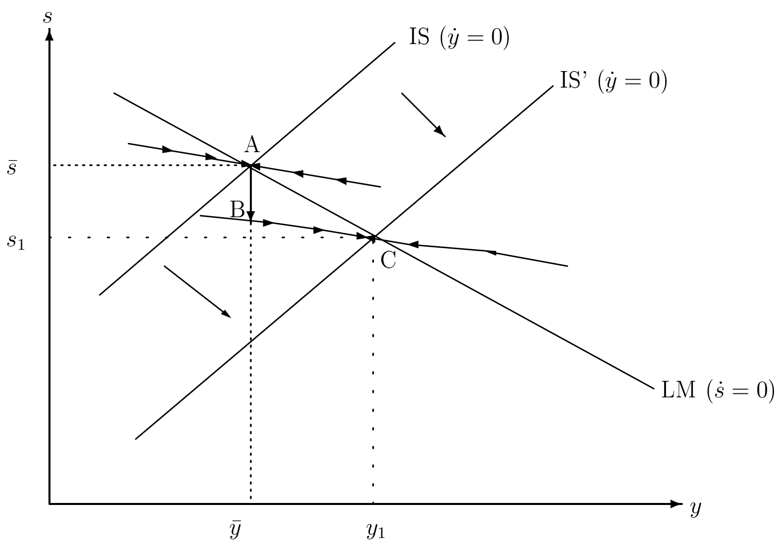
$$\frac{\partial s}{\partial y} = \frac{1}{\mu}.$$



(c) Expansionary monetary policy in the perfect–foresight Mundell–Fleming model. An increase in m will lead to a shift in the LM–curve up to the right. For a given y, higher m implies a higher s (a depreciated exchange rate) which can be seen from the LM–equation. Since the economy is always on a saddlepath, the exchange rate jumps up to the new saddlepath, the exchange rate jumps from point A to point B. At point B, the economy is on a new saddlepath. What happens next? Foreign demand will increase leading to higher output. Output increases and the economy moves along the saddlepath towards the new long–run equilibrium at point C. Total effect: Depreciated currency (an overshooting effect) and higher output (prices are fixed).



(d) Expansionary fiscal policy (a change in α). For given exchange rate, output must increase if $\alpha \uparrow$. The IS–curve shifts down to the right. Output is unchanged initially and the exchange rate jumps down to the new saddlepath. Exchange rate is at point B on the new saddlepath at point B. The lower s reduces foreign demand (but less than α such that there is still excess demand $\mu s < \alpha$). Output will increase along the new saddlepath to new equilibrium. Total effect: An appreciated currency with undershooting and an increase in output.



If $\sigma \to 0$, then the LM–curve is vertical. In this case, fiscal policy will be ineffective.

Thus, fiscal policy is ineffective under floating exchange rates. σ is the weight of foreign prices in the domestic aggregate price level. If foreign prices do not affect the domestic price level, then an increase in α is completely offset by a reduction of foreign demand, there will be no net effect. The inefficiency of fiscal policy in MF–model is a limiting case, not a general case.

(e) This model is also in continuous time. There are two main differences between this model and the model above. In this model output is fixed but the price level is not. In the model above, it is the opposite. Prices are sticky in the letter model but the asset market (the exchange rate market) will react immediately to monetary policy. There is overshooting in both models but the mechanisms are different.

(f) The first two model equations are the same as above (noting that output is constant and remember to replace s with p in the money demand function).

$$\dot{s} = i - i^* \tag{14}$$

$$m = \sigma p + \kappa y - \theta i \tag{15}$$

and then we have equation (4)

$$\dot{p} = \gamma \left(\alpha + \mu \left(s - p\right) - \bar{y}\right) \tag{16}$$

In the long-run equilibrium we have that $\dot{s} = 0$ implying that $i = i^*$ and $p = \bar{p}$. This further implies that

$$m - \bar{p} = \kappa \bar{y} - \theta i^*$$

Subtract this from the money demand function such that

$$p - \bar{p} = \theta(i - i^*)$$

or if we use the UIP relation

$$\dot{s} = \frac{1}{\theta}(p - \bar{p})$$

In equilibrium we know that $\dot{p} = 0$ implying that

$$0 = \gamma \left[\alpha + \mu(\bar{s} - \bar{p}) - \bar{y}\right]$$

which is subtracted from equation (4)

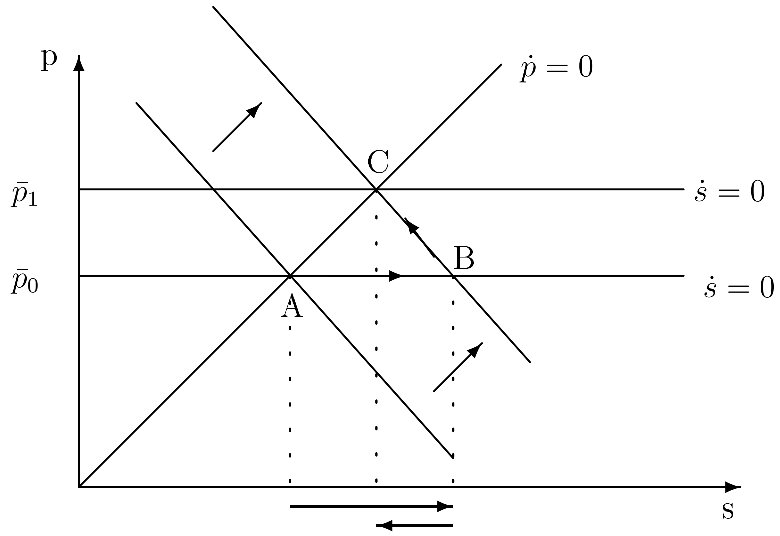
$$\dot{p} = \gamma\mu(s - \bar{s}) - \gamma\mu(p - \bar{p})$$

Now we can summarize the model in matrix form

$$\begin{bmatrix} \dot{s} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{\theta} \\ \gamma\mu & -\gamma\mu \end{bmatrix} \begin{bmatrix} s - \bar{s} \\ p - \bar{p} \end{bmatrix}$$

from which we can calculate the determinant which is $-\frac{\gamma\mu}{\theta}$ indicating that the system has a unique saddlepath. The model can then be illustrated as in the graph below.

(g) Next we are asked to show the effects of expansionary monetary policy. In the long run, the price level will be higher and so will the exchange rate, the exchange rate will depreciate. PPP will ensure that, holding foreign prices constant, the long-run exchange rate will depreciate proportionately. Assume that the economy is in the long-run equilibrium at point A. Expansionary monetary policy will lead to a shift in the saddlepath, then new saddlepath must go throw the new long-run equilibrium at point C where the price level has increased and the exchange rate depreciated. Because prices are sticky the economy cannot jump directly from A to C. Since prices remain fixed in the short-run, the exchange rate jumps to point B initially to get on the new saddlepath. Prices then adjust slowly and the economy moves along the new saddlepath from B to the new long-run equilibrium C. The net effect of the money supply increase is a long-run depreciation corresponding to the distance A to C, with an initial overshooting effect corresponding to A to B. There is an overshooting effect in this model as well.



2. This question relates to the following learning objectives. Knowledge: Describe and explain how macro data releases affect exchange rates and summarize the empirical evidence. Skills: Describe and apply microstructure based models to analyze price determination on the foreign exchange market and summarize the empirical evidence on these models. Competences: Carry out economic analysis related to exchange rate determination, forecasting and international financial management.

   (a) The underlying assumptions of Evans micro-based macro model:

      - There are 2 two countries populated by a continuum of risk-averse agents, indexed by $n \in [0, 1]$; and D risk-averse dealers who act as market makers in the spot market for foreign currency.
      - Two central banks conduct monetary policy by setting short-term nominal in-

terest rates (conditional on inflation, output and real exchange rates).

- The model is simplified such that there is no explicit interdealer trading, the aggregation of private information is not modeled.
- The exchange rate is given by UIP with a risk premium. This risk premium is determined by the requirements of efficient risk sharing, dealers will choose a risk premium such that their holdings of risky currencies at the end of the time period is zero, efficient risk sharing determines the risk premium.
- Each dealer quotes the same price to agents and other dealers and all quotes are functions of the common public information. There is no restriction on dealers to have the same information, they use their heterogeneous information to form their own optimal trading strategies.
- Dealers expect news about the macro economy to affect the expected interest rate differential but there is no assumption about whether or not central banks respond to inflation, output gaps and the exchange rate.
- The timing and information flows are as followed. There is a data release reflecting past macroeconomic activity and based on this information, the central bank then sets the interest rate. Each agent observes the microeconomic environment, they receive private information. Dealers set the price. Given the private information that agents have, they initiate trades with their dealer. Then it is assumed that the aggregate order flow becomes known to all dealers allowing them to revise their quotes.

(b) To show that the aggregate demand for euros can be written as

$$\alpha_t = \alpha_s(\overline{\mathbb{E}}_t^n s_{t+1} - s_t + \hat{r}_t - r_t) + h_t$$

where h_t is the aggregate hedging demand and $\overline{\mathbb{E}}_t^n s_{t+1} = \int_0^1 \mathbb{E}_t^n s_{t+1}\, dn$ we use equations (5), (6) and (7).

Start by aggregating over n agents to find the aggregate hedging demand

$$h_t = \int_0^1 h_t^n dn$$

then insert first (6) and then (7) such that

$$h_t = \int_0^1 h_t^n dn = \alpha_z \left(z_t + \int_0^1 v_t^n \right) = a_z Z_t$$

Next step is to aggregate individual demand for euros

$$\alpha_t = \int_0^1 \alpha_t^n dn.$$

Using (5) we then find that

$$\alpha_t = \alpha_s \left[\int_0^1 \mathbb{E}_t^n s_{t+1} - s_t + \hat{r}_t - r_t \right] + \int_0^1 h_t^n dn$$

implying that

$$\alpha_t = \alpha_s \left[\bar{\mathbb{E}}_t^n s_{t+1} - s_t + \hat{r}_t - r_t\right] + h_t$$

where $\int_0^1 \mathbb{E}_t^n s_{t+1} \equiv \bar{\mathbb{E}}_t^n s_{t+1}$.

(c) We have defined the risk-sharing constraint as

$$\mathbb{E}_t^D \alpha_t = 0.$$

Aggregate demand for euros was defined in (b). Take the dealer expectation of that expression

$$\mathbb{E}_t^D \alpha_t = \mathbb{E}_t^D \left[\alpha_s \left[\bar{\mathbb{E}}_t^n s_{t+1} - s_t + \hat{r}_t - r_t\right] + h_t\right] = 0$$

and rewrite as

$$0 = \alpha_s \mathbb{E}_t^D \left(\bar{\mathbb{E}}_t^n s_{t+1} - s_t\right) + \mathbb{E}_t^D h_t$$

Rewrite (8) as

$$\mathbb{E}_t^D s_{t+1} - \delta_t = s_t - \hat{r}_t + r_t$$

and insert such that

$$0 = \alpha_s \mathbb{E}_t^D \left(\bar{\mathbb{E}}_t^n s_{t+1} - s_{t+1}\right) + \alpha_s \delta_t + \mathbb{E}_t^D h_t.$$

The last step is to use the definition

$$\mathbb{E}_t^D \bar{\mathbb{E}}_t^n s_{t+1} = \mathbb{E}_t^D \bar{\mathbb{E}}_t^n s_{t+1} + \mathbb{E}_t^D s_{t+1} - \mathbb{E}_t^D s_{t+1}$$

which can be rewritten as

$$\mathbb{E}_t^D \bar{\mathbb{E}}_t^n s_{t+1} = \mathbb{E}_t^D s_{t+1} + \mathbb{E}_t^D \left(\bar{\mathbb{E}}_t^n s_{t+1} - s_{t+1}\right)$$

and if we define $\bar{\mathbb{E}}_t^n s_{t+1} - s_{t+1} \equiv -s_{t+1}^e$ we have that

$$0 = -\alpha_s \mathbb{E}_t^D s_{t+1}^e + \alpha_s \delta_t + \mathbb{E}_t^D h_t$$

and finally solving for the risk premium we find

$$\delta_t = \mathbb{E}_t^D \left[s_{t+1}^e - \frac{1}{\alpha_s} h_t\right]$$

(d) The question states that the exchange rate in this model is given by

$$s_t = (\hat{r}_t - r_t) + \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i f_{t+i} + \tfrac{1}{\alpha_s} \mathbb{E}_t^D \sum_{i=0}^{\infty} \rho^i h_{t+i} - \tfrac{1}{\rho} \mathbb{E}_t^D \sum_{i=1}^{\infty} \rho^i s_{t+i}^e.$$

The following predictions can be drawn from the equation:

- Compared to standard macro models, this model includes not only future discounted fundamentals and a risk premium but specifies a model for the risk premium where agents hedging demand and dealers expectation of the average agent expectation of the future exchange rate.

- We find that the actual spot exchange rate is a function only of information available to dealers at the time they quote prices. This information set includes contemporaneous interest rates but not other contemporaneous fundamentals.
- As mentioned above, the risk premium incorporates dealers' estimates of aggregate hedging demand and agents' forecast errors.
- All these factors can, potentially, be a source of variation in the spot rate even if dealers expect current and future monetary policy to be unchanged.
- Standard macro models suggest that spot rates depend on current and expected future fundamentals whereas dealers expectation of the agents' average forecast errors affect the spot rate via their implication for risk-sharing in this model.

(e) Empirical evidence strongly supports the idea that order flows significantly affect quoted prices and exchange rate returns. There is a strong positive contemporaneous correlation between daily changes in the price of FX and interdealer order flow. This positive correlation is robust to different forms of interdealer trading and appears across a wide cross-section of currencies. Interbank order flows explain a large portion of the variance in spot exchange rates using single currency regressions. The explanatory power is also increasing when allowing for order flows in a particular currency to also affect other currencies. Compared to empirical studies based on standard macro models we also find that interdealer order flows explain more than that found for any other macroeconomic or financial variables. The impact of order flows on exchange rates may depend on trading volume, the empirical evidence is not conclusive yet.

The contemporaneous relationship between spot rate changes and order flow applies also to customer order flows. Disaggregated customer order flows (disaggregated by customer type) have more explanatory power for exchange rate returns than the aggregate flows received by individual banks. Disaggregated customer order flows account for less of the variation in exchange rate returns compared to aggregate interdealer order flows, but the explanatory power of customer and dealers flows are comparable at lower frequencies.

Finally, empirical evidence suggests that macro news announcements do explain parts of the agent initiated trades, new information about the macro economy leads agents to change their demand for currencies. These news initiated order flows carry information to dealers leading them to revise their quotes.